10.3 Logarithmic Functions

I. What is a Logarithmic Function?

What is the inverse operation of…

Operation	**Inverse operation**	***Example.*** Solve for x
Addition		$5 = x + 3$
Multiplication		$12 = 3x$
Squaring		$121 = x^2$
Exponential		$100000 = 10^x$

Definition **- Logarithmic Function:**

For $x > 0$ and $b > 0$, $b \neq 1$,

$b^y = x$ is equivalent to $y = \log_b x$

This means that finding $\log_b x$ is equivalent to asking the question:
"$What\ power\ of\ base\ b\ is\ needed\ to\ get\ the\ value\ x?$"

Exponential Form: $\boldsymbol{b^y = x}$
Logarithmic Form: $\boldsymbol{y = log_b x}$

Key point: The logarithm, y, is the exponent. The logarithmic form allows us to isolate the exponent.

Changing forms

Example 1: Write each equation in its equivalent form and evaluate the variable.

a. $4 = \log_2 x$ b. $3 = \log_b 125$ c. $y = \log_2 16$

Example 2: Write each equation in its equivalent logarithmic form.

a. $10^2 = x$ b. $b^3 = 64$ c. $e^y = 23$

Evaluation Logarithms

Example 3: Evaluate the logarithm by rewriting it in exponential form.

a. $\log_2 32$ b. $\log_5\left(\frac{1}{25}\right)$ c. $\log_{64} 8$

II. Basic Logarithmic Properties

Basic Logarithmic Properties Involving 1

1. $\log_b b = 1$
2. $\log_b 1 = 0$

Example 4: Evaluate

a. $\log_{123} 123$ b. $\log_{19} 1$

Inverse Properties of Logarithms

For $b > 0$ and $b \neq 1$,

1. $\log_b b^x = x$
2. $b^{\log_b x} = x$

Example 5: Evaluate

a. $\log_7 7^5$ b. $8^{\log_8 9}$

Common Logarithms

Definition: A common logarithm is a base 10 logarithm.

$f(x) = \log_{10} x$ is commonly written as $f(x) = \log x$

Example6: Compute each of the following (no calculator for *a* and *b*.)

a. $\log 100000$ b. $\log\left(\frac{1}{100}\right)$ c. $\log\left(\frac{3}{7}\right)$ d. $\log(-5)$

Solving Basic Logarithmic Equations

To solve a Logarithmic equation, convert to an exponential equation and solve.

Example Solve these equations

a. $\log_3(x+1) = 4$

b. $\log(3x+10) = 2$

III. Graphs of Logarithmic Functions

Remember… The inverse of a function reverses the x and y coordinates.

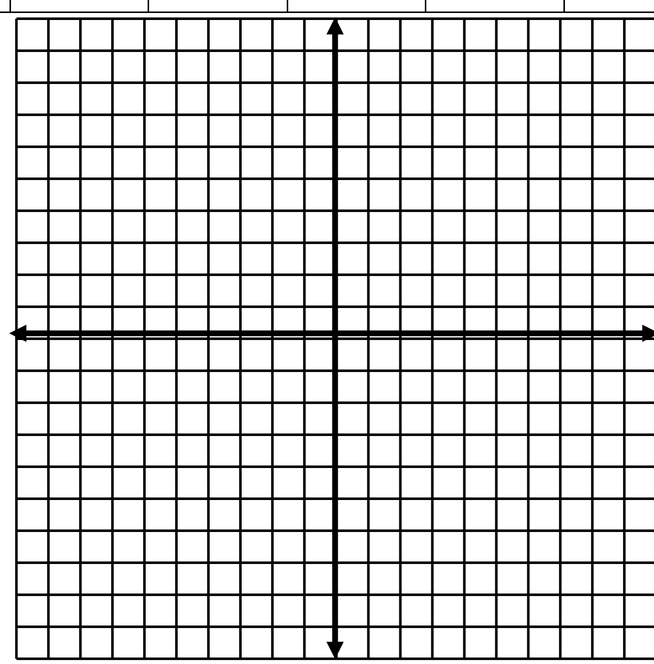
Example 7: Graph $g(x) = \log_2 x$ and its inverse on the same axes.

1. Write the inverse, $g^{-1}(x)$
2. Make a table of values for $g^{-1}(x)$:

x						
$g^{-1}(x) =$						

3. Reverse the domain and range values from the table to find the points of

x						
$g^{-1}(x) =$						

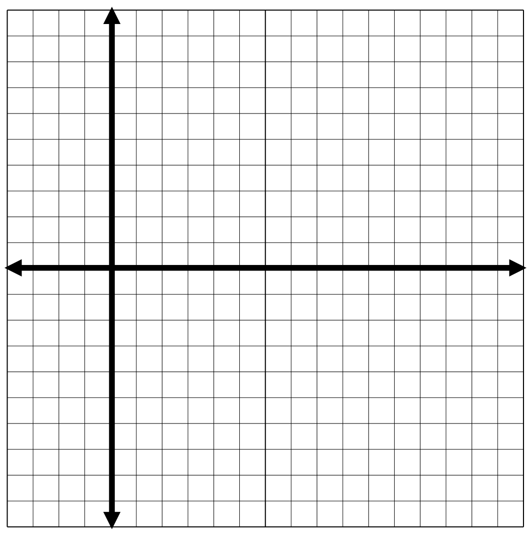


Steps for graphing a simple logarithmic function:

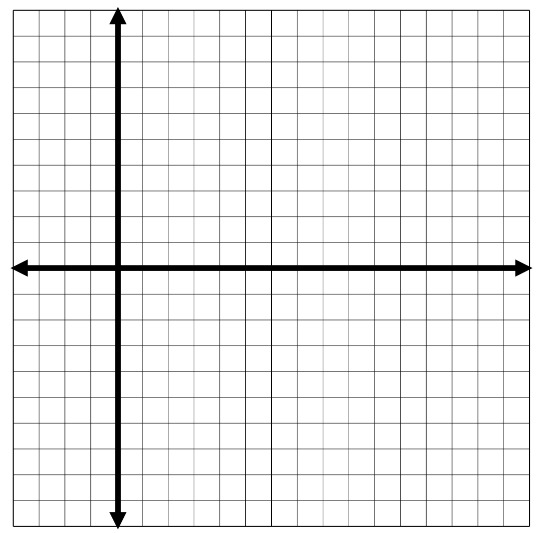
1. Write the function in terms of x and y. (e.g. $y = \log_b x$)
2. Change the function into its equivalent exponential form for x as a function of y (e.g. $x = b^y$)
3. Find (x, y) points for this exponential function in "reverse".
4. Graph these points

Example Graph by hand.

a. $f(x) = \log_3 x$



b. $g(x) = \log_{1/2} x$



Characteristics of Logarithmic Functions of the form $\boldsymbol{F(x) = log_b x}$

1. Domain: All positive real numbers $(0, \infty)$.
   Range: All Real numbers $(-\infty, \infty)$
2. All logarithmic functions of the form $f(x) = \log_b x$ pass through the point $(1,0)$
3. If $b > 1$, the graph goes up to the right and is an ***increasing*** function.
4. If $0 < b < 1$, the graph goes down to the right and is a ***decreasing*** function.
5. The graph of $f(x) = \log_b x$ approaches but does not touch the ***y-axis***.
   The y-axis or x=0 is a vertical asymptote.

Natural Logarithms

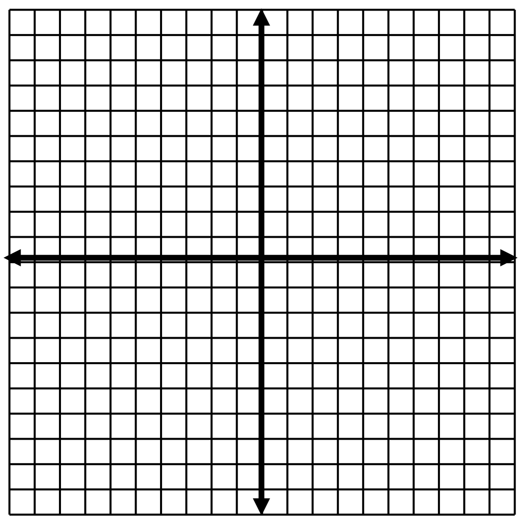
<u>Definition</u>: A natural logarithm is base *e*.

$$\boldsymbol{f(x) = log_e x} \text{ is commonly written } \boldsymbol{f(x) = \ln x}.$$

<u>Example</u> Compute the natural logarithms using your calculator, then check your answer using e^x.

a. $\ln 9$ b. $\ln 5$ c. $\frac{\ln(2)}{3}$

<u>Example:</u> Use your calculator to graph $h(x) = \ln x$ and sketch it here.



***<u>Example</u>*: Sketch a graph of the function and find the domain and the range**

a. $f(x) = \ln x$ b. $g(x) = \ln(x + 5)$

b. $h(x) = \ln x + 5$